

國立嘉義大學九十三年度轉學生招生考試試題

科目：離散數學

(請將答案寫在答案卷上)

1. Mathematical Induction

(1) Describe the *Well-Ordering Principle*. (10 %)

(2) Consider the following four equations: (10%)

$$\begin{aligned} 1 &= 1 \\ 2+3+4 &= 1+8 \\ 5+6+7+8+9 &= 8+27 \\ 10+11+12+13+14+15+16 &= 27+64 \end{aligned}$$

Conjecture the general formula suggested by these four equations and prove your conjecture.

2. (1) Solve the following recurrence relations. You should write the detail computing processes.

$$a_{n+2} - 6a_{n+1} + 9a_n = 3(2^n) + 7(3^n), \text{ where } n \geq 0 \text{ and } a_0 = 1, a_1 = 4. \quad (10\%)$$

(2) Find a recurrence relation and give initial conditions for the number of bit strings of length n that do *not* have two consecutive 0's. How many such bit strings are there of length eight. (10%)

3. Consider the expression: $((A+B)*C)-D*(E-F)/G$

(1) Give a binary tree to represent the expression. (5%)

(2) Write the prefix form of the expression. (5%)

(3) Write the postfix form of the expression. (5%)

4. Let $A = 2, B = 3, C = 4, D = 5, E = 6, F = 7$. Find the value of following expressions.

(1) prefix expression: $+ * + - AB - CDEF$ (5%)

(2) postfix expression: $AB + CDE - - * F +$ (5%)

5. Consider the language $L = \{w \mid w \in \{a, b\}^*, w \text{ does not contain substring } aaa\}$.

(1) Please give a grammar that specifies L . (5%)

(2) Give an automaton to accept L . (5%)

6. Find the shortest distance from v_0 to all other vertices in the following graph. (Hint: Use Dijkstra's algorithm) (20%)

