國立嘉義大學九十五學年度
光電暨固態電子研究所碩士班招生考試試題

科目：工程數學

1. Solve $\begin{cases} \dot{x} = 2x + 3y + 2e^{2t} \\ \dot{y} = x + 4y + 3e^{2t} \end{cases}$, $x(0) = -\frac{2}{3}$, $y(0) = \frac{1}{3}$ (10%)

   Note that $\dot{x}$ and $\dot{y}$ denote $\mathrm{d}x/\mathrm{d}t$ and $\mathrm{d}y/\mathrm{d}t$ respectively.

2. Solve $xy' + y = xy^3$, where y' means $\mathrm{d}y/\mathrm{d}x$. (10%)

3. Find the eigenvalues and the corresponding eigenvectors of the matrix

$$\mathbf{A} = \begin{bmatrix} \frac{3}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{3}{4} & 0 \\ -\frac{1}{4} & -\frac{1}{4} & \frac{2}{4} \end{bmatrix}. \qquad (10\%)$$

4. Kirchhoff's equation for the analogous electrical circuit is as follows:

$$L\ddot{q}(t) + R\dot{q}(t) + \frac{1}{C}q(t) = E\sin\omega_0 t,$$

   where L is inductance, R is resistance, C is capacitance, q is charge, E is the amplitude of the applied voltage，ω_0 is the frequency of the applied voltage, and $\dot{q}$ and $\ddot{q}$ denote $\mathrm{d}q/\mathrm{d}t$ and $\mathrm{d}^2q/\mathrm{d}t^2$ respectively.

   Find the general solution of $q(t)$. (20%)

5. Rewrite $\sin^3 x$ in terms of $\cos x$, $\sin x$, $\cos 3x$, and $\sin 3x$. (10%)

   Solve the ordinary differential equation: $\frac{\mathrm{d}y}{\mathrm{d}x} = 12\sin^3 x$, $y(0) = -8$. (10%)

   【Hint】Try de Moivre's formula $(\cos\theta + \mathrm{i}\sin\theta)^3 = \cos 3\theta + \mathrm{i}\sin 3\theta$.

6. Find the conditions on η such that the simultaneous equations (10%)

$$\begin{cases} x + y + z = 1 \\ x + 2y + 4z = \eta \\ x + 4y + 10z = \eta^2 \end{cases}$$

   have solutions.

7. Find the quadratic forms for kinetic energy $E_\mathrm{k} = \left(\frac{1}{2}m\right)(v_1^2 + \mu v_2^2 + v_3^2)$ and potential energy $E_\mathrm{p} = \left(\frac{1}{2}k\right)[(x_2 - x_1)^2 + (x_3 - x_2)^2]$ of oscillation of three particles of masses $m, \mu m, m$ connected in that order in a straight line by two equal light springs of force constant k, i.e. in the forms of

$$E_\mathrm{k} = \mathbf{v}^\mathrm{T}\mathbf{A}\mathbf{v} = \begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix} \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{12} & A_{22} & A_{23} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \text{ and}$$

$$E_\mathrm{p} = \mathbf{x}^\mathrm{T}\mathbf{B}\mathbf{x} = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{12} & B_{22} & B_{23} \\ B_{13} & B_{23} & B_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}. \quad (10\%)$$

   Solve $\det(\mathbf{B} - \omega^2\mathbf{A}) = 0$ to find the normal frequencies ω. (10%)