國立嘉義大學 100 學年度
應用數學系碩士班（乙組）招生考試試題

科目：基礎數學

說明：(1)本試題有微積分、線性代數二大部分，各佔 50 分。
(2)本試題為計算、證明題，請標明每部分的題號，同時將過程作答在「答案卷」上。

一、微積分部分

1. (a) Find $\lim_{x\to\infty}(1-\frac{1}{x})^{x}$=? (5%) (b) Evaluate $\lim_{x\to 3}(\frac{x}{x-3}\cdot\int_{3}^{x}\frac{\sin t}{t}dt)$ =? (10%)

2. Evaluate $\int_{-1}^{2}\frac{1}{x^3}dx$=? (10%)

3. Find the following derivatives $\frac{dy}{dx}$, if exists. (15%)
(a) $y=\ln(x)$ (b) $y=x^{\sin x}$

4. Find $\sum_{n=1}^{\infty}\frac{\sqrt{n+1}-\sqrt{n}}{\sqrt{n^2+}}$. (10%)

二、線性代數部分

1. Find the eigenvalues and eigenvectors of a matrix $A=\begin{bmatrix}1&3&4&2\\4&1&3&2\\2&2&1&0\end{bmatrix}$. (15%)

2. Prove: A set $\{v_1, v_2, \cdots, v_n\}$ of vectors in a vector space V is linearly dependent if and only if some v_i is a linear combination of the others. (10%)

3. If A is nonsingular. Prove that $|A^{-1}|=|A|^{-1}$. (8%)

4. Given $A=\begin{bmatrix}1&2&3\\2&5&1\\5&12&5\end{bmatrix}$. Find all vectors $u=(x_1,x_2,x_3)^T$ such that $Au=0$, where 0 is a zero vector. (7%)

5. (a) Find the rank of $A=\begin{bmatrix}1&2&3\\2&1&0\\2&1&3\\1&4&2\end{bmatrix}$. (5%)

(b) Find the rank of $B=\begin{bmatrix}1&2&0&1\\2&6&3&3\\3&10&5&1\end{bmatrix}$. (5%)