國立嘉義大學 99 學年度

應用數學系碩士班（乙組）招生考試試題

科目：基礎數學

說明：(1)本試題有微積分、線性代數二大部分，各佔 50 分。

(2)本試題為計算、證明題，請標明每部分的題號，同時將過程作答在「答案卷」上。

一、微積分部分：(50 分)

1. (a) Find $\lim_{x\to\infty} x\log\left(\frac{x+1}{x-1}\right)$. (8%) (b) Find $\lim_{x\to 0}(1+ax)^{1/x}$. (7%)

2. Find $\int_0^{\infty} x^n e^{-x}dx$. (10%)

3. Find the following derivatives dy/dx, if exists. (15%)

(a) $y=\int_{\cot x}^{\ln x^3}\ln t\,dt.$ (b) $\ln(xy^2)+2x=y^2+xy+2.$

4. Let $f(x)=xe^x$. (a) Expand $f(x)$ in a power series. (b) Integrate the power series in part (a) and show that $\sum_{n=1}^{\infty}\frac{1}{n!(n+2)}=\frac{1}{2}$. (10%)

二、線性代數部分：(50 分)

1. Calculate the rank of $A=\begin{bmatrix} 1 & 4 & 6\\ 4 & -1 & 2\\ -7 & 6 & 2\\ 2 & 8 & 13\end{bmatrix}$. (7%)

2. Find the inverse of $B=\begin{bmatrix} -1 & 3 & 0\\ 0 & 2 & 1\\ 1 & 0 & 4\end{bmatrix}$. (10%)

3. Let $A=\begin{bmatrix} 1 & 0 & 0\\ 0 & \frac{2}{3} & \frac{\sqrt{2}}{3}\\ 0 & \frac{\sqrt{2}}{3} & \frac{2}{3}\end{bmatrix}$. Prove that A is an idempotent matrix. (8%)

4. Let A be a $n\times n$ symmetric matrix with eigenvalues $\lambda_1,\lambda_2,\cdots,\lambda_n$. (10%)

(a) Prove that $|A|=\prod_{i=1}^{n}\lambda_i$.

(b) Prove that $tr(A)=\sum_{i=1}^{n}\lambda_i$.

5. Let $A=\begin{bmatrix} 0 & 0 & -2\\ 1 & 2 & 1\\ 1 & 0 & 3\end{bmatrix}$. Find an invertible matrix P and a diagonal matrix D such that $A=PDP^{-1}$. (15%)