

國立嘉義大學九十七學年度  
土木與水資源工程學系碩士班招生考試試題

科目：工程數學

說明：如條件不足，請自行假設。

1. Solve the following ordinary differential equation: (20%)

$$y'' + y' - 2y = 6e^x$$

2. Solve the following ordinary differential equations using Laplace Transformation: (20%)

$$\textcircled{1} y(t) = 1 + \int_0^t y(\tau) d\tau$$

$$\textcircled{2} y(t) = 1 + y'(t) \quad , y(0) = 1$$

3. Solve the following ordinary differential equation: (20%)

$$y^{(4)} + 3y = e^{2x}$$

4. Define  $\nabla = \frac{\partial}{\partial x} \bar{i} + \frac{\partial}{\partial y} \bar{j} + \frac{\partial}{\partial z} \bar{k}$  as an operator, a function  $F(x, y, z) = x^2 + y^2 + z^2$ ,

and a vector  $\bar{v} = x^3 \bar{i} + y^3 \bar{j} + z^3 \bar{k}$  are known. Calculate results of the following questions: (20%)

$$\textcircled{1} \text{ gradient } \nabla F = ? \quad \textcircled{2} \text{ divergence } \nabla \cdot \bar{v} = ? \quad \textcircled{3} \text{ curl } \nabla \times \bar{v} = ?$$

$$\textcircled{4} \nabla \nabla F = ? \quad \textcircled{5} \nabla \cdot \nabla F = ? \quad \textcircled{6} \nabla \times \nabla F = ?$$

$$\textcircled{7} \nabla \nabla \bar{v} = ? \quad \textcircled{8} \nabla \cdot \nabla \cdot \bar{v} = ? \quad \textcircled{9} \nabla \times \nabla \times \bar{v} = ? \quad \textcircled{10} \nabla \cdot \nabla \times \bar{v} = ?$$

5. For the following matrix, (a) find the eigenvalues and eigenvectors;

(b) diagonalize the matrix  $\mathbf{A}$  by using eigenvectors. (20%)

$$\mathbf{A} = \begin{bmatrix} -3 & 2 \\ -10 & 6 \end{bmatrix}$$