國立嘉義大學 99 學年度

數理教育研究所碩士班(甲組)招生考試試題

科目：微積分

1. Find the limit for each: (10%)

   (a) $\lim\limits_{x\to\infty}(\sqrt{4x^2+3x}-2x)=$ _____ (b) $\lim\limits_{x\to 0}\dfrac{x^3-x^2}{e^x+e^{-x}-2}=$ _____

2. Evaluate：(20%)

   (a) $\int \sec^3 x\,dx=?$ (b) $\int_{-\infty}^{\infty}\dfrac{e^x}{1+e^{2x}}dx=?$

3. Let R be the solid region bounded by the graphs $y=\dfrac{1}{4}x^2$, $x=0$ and the line $y=1$. Find the volume of the solid generated when R is revolved about (a) the x-axis and (b) the line y=2 respectively. (20%)

4. Evaluate each of the following limits, if it exists. (20%)

   (a) $\lim\limits_{x\to 1}\dfrac{x^3+2x-3}{x^2-4x+3}$ (b) $\lim\limits_{x\to 3}\dfrac{\sqrt{12-x}-3}{\sqrt{4-x}-1}$

5. (a) Let $f(x)=(x^4+1)^{2010}$. Find $f''(x)$. (10%)

   (b) Suppose that $g'''(a)$ exists. Find $\lim\limits_{h\to 0}\dfrac{g(a+h)-g(a-h)-2hg'(a)}{h^3}$. (10%)

6. Suppose that $f(1)=3$ and $f'(x)\le 3, \forall x \in R.$ How large can $f(4)$ possibly be? (10%)