國立嘉義大學九十七學年度
光電暨固態電子研究所碩士班招生考試試題

科目：工程數學

Write only either “True” or “False”, in answering the following true-false questions 1-10
（1-10 題寫下“True”或是“False”. 一題 4 分，答錯倒扣 2 分。）

1. $x(t)=\mathrm{e}^{\beta t}(c_1\cos\omega t+c_2\sin\omega t)$ is the general solution of the ordinary differential equation $\frac{\mathrm{d}^2x}{\mathrm{d}t^2}-2\beta\frac{\mathrm{d}x}{\mathrm{d}t}+(\omega^2+\beta^2)x=0$, where c_1 and c_2 are arbitrary numbers. True or false?

2. $y(x)=\cosh^{-1}x$ is a particular solution of the ordinary differential equation $(x^2+1)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+x\frac{\mathrm{d}y}{\mathrm{d}x}=0$. True or false?

3. If vectors $\vec{\mathbf{a}}$, $\vec{\mathbf{b}}$, and $\vec{\mathbf{c}}$ are not coplanar and we have $\vec{\mathbf{a}}'=\frac{\vec{\mathbf{b}}\times\vec{\mathbf{c}}}{\vec{\mathbf{a}}\cdot(\vec{\mathbf{b}}\times\vec{\mathbf{c}})}$, then $\vec{\mathbf{a}}'\cdot\vec{\mathbf{b}}=\vec{\mathbf{a}}'\cdot\vec{\mathbf{c}}$ holds. True or false?

4. If vectors $\vec{\mathbf{a}}$, $\vec{\mathbf{b}}$, and $\vec{\mathbf{c}}$ are not coplanar and we have $\vec{\mathbf{a}}'=\frac{\vec{\mathbf{b}}\times\vec{\mathbf{c}}}{\vec{\mathbf{a}}\cdot(\vec{\mathbf{b}}\times\vec{\mathbf{c}})}$, and $\vec{\mathbf{b}}'=\frac{\vec{\mathbf{c}}\times\vec{\mathbf{a}}}{\vec{\mathbf{a}}\cdot(\vec{\mathbf{b}}\times\vec{\mathbf{c}})}$, then $\vec{\mathbf{a}}'\cdot\vec{\mathbf{a}}=\vec{\mathbf{b}}'\cdot\vec{\mathbf{b}}$ holds. True or false?

5. If a matrix $\mathbf{A}$ is orthogonal (i.e., $\mathbf{A}^{\mathrm{T}}=\mathbf{A}^{-1}$), the determinant must be $|\mathbf{A}|=-1$. True or false?

6. The matrix $\mathbf{A}=\begin{bmatrix}\sqrt{3} & -\sqrt{2} & -\sqrt{3}\\ 1 & \sqrt{6} & -1\\ 2 & 0 & 2\end{bmatrix}$ is Hermitian. True or false?

7. If the inner product of two functions is defined as $\langle f|g\rangle=\int_{-\pi}^{\pi}f^*(x)\,g(x)\,\mathrm{d}x$ (where * denotes the complex conjugate), then f and g are said to be orthogonal to each other when $\langle f|g\rangle=0$. True or false?

8. If the inner product of two functions is defined as $\langle f|g\rangle=\int_{-\pi}^{\pi}f^*(x)\,g(x)\,\mathrm{d}x$ (where * denotes the complex conjugate), then we must have $\langle f|f\rangle=1$ for any function f. True or false?

9. If the Fourier transform $\tilde{f}(k)$ of any function $f(x)$ is defined as $\tilde{f}(k)=\frac{1}{2\pi}\int_{-\infty}^{\infty}f(x)\mathrm{e}^{-ikx}\mathrm{d}x$, and δ denotes the Dirac delta function, then the Fourier transform of $\sin k_0x$ is given by $\sqrt{2\pi}\,\delta(k-k_0)$. True or false?

10. If the Fourier transform $\tilde{f}(k)$ of any function $f(x)$ is defined as $\tilde{f}(k)=\frac{1}{2\pi}\int_{-\infty}^{\infty}f(x)\mathrm{e}^{-ikx}\mathrm{d}x$, and δ denotes the Dirac delta function, then the Fourier transform of $\cos k_0x$ is given by $\frac{\sqrt{2\pi}}{2}[\delta(k-k_0)+\delta(k+k_0)]$. True or false?

（11-13 題為計算題，必須寫下計算過程及必要之說明。每題 20 分）

11. The falling parachutist (mass m) encounters the air resistance $-bv^2$, opposing the force of the gravitational attraction, mg, of the earth. Initial condition is given by $v=0$ at time $t=0$. Solve Newton’s equation of motion $m\frac{\mathrm{d}v}{\mathrm{d}t}=mg-bv^2$. Find the velocity as a function of time, in terms of $v_0=\sqrt{mg/b}$ and $T=\sqrt{m/(gb)}$.

12. The action of the control mechanism on a particular system for an input $f(t)$ is described, for $t\ge0$, by the coupled first-order equations $\frac{\mathrm{d}y}{\mathrm{d}t}+4z=f(t)=H(t)$, $\frac{\mathrm{d}z}{\mathrm{d}t}-2z=\frac{\mathrm{d}y}{\mathrm{d}t}+\frac{1}{2}y$, where H denotes the Heaviside unit step function. Find the response $y(t)$ of the system, given that $y(0)=1$ and $z(0)=0$.

13. Find the complex Fourier expansion of $f(x)$, which is given by
$$f(x)=\sum_{n=1}^{\infty}\frac{-(-1)^n}{n\pi}\sin(2n\pi x)=\frac{1}{\pi}\sin 2\pi x-\frac{1}{2\pi}\sin 4\pi x+\frac{1}{3\pi}\sin 6\pi x-\ldots$$