國立嘉義大學 99 學年度

行銷與運籌研究所碩士班(乙組)招生考試試題

科目：統計學

※可使用計算機

1. Two continuous random variables X and Y have a joint probability density function of the form

$$f_{XY}(x, y) = \begin{cases} a(xy + x + y), & 0 \le x \le 3,\ 0 \le y \le 2, \\ 0, & elsewhere \end{cases}$$

   (1) Find the value of a. （5%）
   (2) Find the marginal probability density functions $f_X(x)$ and $f_Y(y)$. （10%）
   (3) Find the conditional probability density functions $f(x/y)$ and $f(y/x)$. （10%）

2. Manufactured machines are tested for defects. Hopefully, the machines should fail (F) if a defect (D) is identified. Similarly, a machine should not fail (F^c) if the test indicates no defect (D^c). However, it is possible for a given machine having no defect to fail the test, and for a machine having a defect to pass the test. Given the following probabilities,

   $P(D^c) = 0.8$,
   $P(F|D) = 0.9$,
   $P(F|D^c) = 0.05$,

   (1) What is the failure probability, $P(F)$? （10%）
   (2) What is the probability of $P(D|F)$? （10%）
   (3) What is the probability of $P(D^c|F^c)$? （5%）

3. A sample of test scores of statistics is given as follows.

   77 95 87 38 43 52 84 41 91 47

   (1) Construct a stem-and-leaf plot. Is the distribution symmetric, positively skewed, negatively skewed, or binomial? (4%)
   (2) Find the sample mean and 10% trimmed mean. (6%)
   (3) Find the median, first sample quartile and third sample quartile. (6%)
   (4) Compute the 10 deviations from the mean, and verify that they have a sum of zero. (3%)
   (5) Compute the sample variance and the standard deviation. (6%)

4. How significant is the earned run average (ERA) for pitchers in the major league baseball in determining the number of wins for the team? To investigate, consider the number of wins and the earned run averages for the National League teams at the end of the 1990 season.

Team	Wins	ERA
Atlanta	65	4.6
Chicago	77	4.3
Cincinnati	91	3.4
Houston	75	3.6
Los Angeles	86	3.7
Montreal	85	3.4
New York	91	3.4
Philadelphia	77	4.1
Pittsburg	95	3.4
St. Louis	70	3.9
San Diego	75	3.7
San Francisco	85	4.1

   (1) Construct a scatter plot of the data. Does it appear that a linear model would describe the relationship between the number of wins and the ERA. (3%)
   (2) Calculate and **interpret** the estimates of β_0 and β_1 for the linear model $y = \beta_0 + \beta_1 x + \varepsilon$. (10%)
   (3) Calculate and **interpret** the value of r^2 for this data set. (5%)
   (4) What can you say about a team with ERA equal to 1.0? (3%)
   (5) Set up the ANOVA table. (4%)