

國立嘉義大學九十七學年度  
生物機電工程學系碩士班招生考試試題

科目：工程數學

(※禁止使用計算機)

1. A function  $y = y(x)$  satisfies the 1<sup>st</sup>-order differential equation:

$$2xy \frac{dy}{dx} + 4x + 3y^2 = 0$$

(a) Does the differential equation satisfy the “Condition of Exactness”? (10%)

(b) Solve the differential equation using the method of integration factor. (10%)

2. Consider the differential equation  $\rho \frac{\partial u}{\partial t} = -\sum_{n=0}^N C_n e^{in\omega t} + \mu \left( \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right)$ , where  $n, \rho, \mu, C_n$ , and  $\omega$  are

arbitrary constants,  $\sqrt{-1} = i$ , and  $u(r, t) = \sum_{n=0}^N v_n(r) e^{in\omega t}$ .

(a) Reduce the equation given above to Bessel’s equation. (10%)

(b) The general solution of the Bessel’s equation given in part (a) is

$$v_n(r) = A_n J_0 \left( \beta \frac{r}{a} n^{1/2} i^{3/2} \right) + B_n Y_0 \left( \beta \frac{r}{a} n^{1/2} i^{3/2} \right) + \frac{i C_n}{\rho n \omega}$$

where  $A_n, B_n, a$ , and  $\beta$  are arbitrary constants. The boundary conditions are

$$v_n = 0 \quad \text{when } r = a,$$

$$\frac{\partial v_n}{\partial r} = 0 \quad \text{when } r = 0.$$

Find  $A_n$  and  $B_n$ . (10%)

3. Consider the following linear equation with 3 unknowns  $x, y$  and  $z$ :

$$\begin{cases} a y + z = b \\ a x + b z = 1 \\ a x + a y + 2 z = 2 \end{cases}$$

Determine constants  $a$  and  $b$  (if any) such that the system possesses the following:

(a) A unique solution. (5%)

(b) A one-parameter solution. (5%)

(c) A two-parameter solution. (5%)

(d) No solution. (5%)

4. Evaluate the followings:

(a) the Laplace transform of the piecewise function  $f(t) = \begin{cases} 2, & 0 \leq t < 3 \\ -2, & t \geq 3 \end{cases}$ . (7%)

(b) the inverse Laplace transform of  $G(s) = \frac{1}{s^2(s^2 + b^2)}$ . (6%)

(c) the inverse Laplace transform of  $H(s) = \ln \frac{s^2 + b^2}{s^2}$ . (7%)

5. Given a matrix  $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ , construct an orthogonal matrix  $R$  from  $A$ . (20%)