

國立嘉義大學 100 學年度

資訊工程學系碩士班 (甲組) 招生考試試題

科目：離散數學

- Use mathematical induction to show that $11^{n+2} + 12^{2n+1}$ is divisible by 133 for any natural number n . (10%)
- Given $(1+x)^n = \sum_{i=0}^n C_i^n x^i$, please show that $\sum_{i=1}^n i \times C_i^n = n \times 2^{n-1}$. (10%)
- With each step you take when climbing a staircase, you can move up either one stair or two stairs. As a result, you can climb the entire staircase taking one stair at a time, taking two at a time, or taking a combination of one- and two-stair increments. For each integer $n \geq 1$, if the staircase consists of n stairs, let c_n be the number of different ways to climb the staircase.
 - Compute c_1, c_2, c_3, c_4 , and c_5 . (5%)
 - Find a recurrence relation for $c_1, c_2, c_3, c_4, c_5, \dots$. (5%)
 - Find an explicit formula for c_n . (10%)
- Find the number of integer solutions to $c_1 + c_2 + c_3 + c_4 + c_5 = 10$ where $0 \leq c_1, 1 \leq c_2, 0 \leq c_3, 2 \leq c_4$, and $0 \leq c_5$. (10%)
- Determine whether each of the following statements is true or false. Briefly explain your answer. (10%)
 - $\phi \subseteq \phi$, where ϕ means an empty set
 - $\phi \in \{\phi\}$
 - $\{\} \notin \{\phi\}$
 - $\{a,b\} \in \{\{\{a,b\}\}, a, b\}$
 - $\{a,\{\}\} \subseteq \{\{a\},\{\},\{a,\{\}\}\}$
- Use K-map to simplify the Boolean function $F = wxyz + wxyz' + wxy'z' + wxy'z + w'xyz + w'xy'z$. (10%)
- Show that $(p \rightarrow q) \rightarrow r$ and $p \rightarrow (q \rightarrow r)$ are not logically equivalent. (10%)
- If we arbitrarily place 7 identical black balls and 5 identical white balls in 4 numbered boxes, what is the probability that each box contains at least one ball of each color? (10%)
- Please determine whether the following functions from Z to Z are invertible (one-to-one and onto) functions or not?
 - $f(x) = \lceil x/4 \rceil + 1$ (4%)
 - $f(x) = x+3$ (3%)
 - $f(x) = x^2 + 1$ (3%)