國立嘉義大學九十七學年度
應用數學系碩士班（乙組）招生考試試題

科目：機率統計

說 明：(1) 本試題有機率、統計二大部分，各佔 50 分。
(2) 本試題為計算、證明題，請標明每部分的題號，同時將過程作答在「答案卷」上。
(3) 統計部分，計算過程如有需要常態曲線下之面積數值，請參考題目後面之附表。

一、機率部分：50 分

1. Let $X_j,\ j=1,2,\cdots,n$ be independent r.v.'s distributed as $N(\mu,\sigma^2)$ and set

$$X=\sum_{j=1}^{n}\alpha_j X_j,\quad Y=\sum_{j=1}^{n}\beta_j X_j,$$

where the α's and β's are constants. Then
(a) Find the p.d.f.'s of the r.v.'s X, Y. (8%)
(b) Under what conditions on the α's and β's are the r.v.'s X and Y independent? (7%)

2. Let X be an r.v. with $E(X)=0$ and $Var(X)=\sigma^2$. Show that
(a) $P(X\ge x)\le\dfrac{\sigma^2}{\sigma^2+x^2}$, $x>0$. (5%)
(b) $P(X\ge x)\ge\dfrac{x^2}{\sigma^2+x^2}$, $x>0$. (5%)

3. Suppose that a random system of police patrol is devised so that a patrol officer may visit a given location $Y=0,1,2,3,\cdots$ times per half-hour period and that the system is arranged so that each location is visited on an average of once per time period. Assume that Y possesses, approximately, a Poisson probability distribution.
(1) Calculate the probability that the patrol officer will miss a given location during a half-hour period. (8%)
(2) What is the probability that it will be visited at least once? (7%)

4. Suppose that the joint density of X and Y is given by
$f(x,y)=\dfrac{e^{-x/y}e^{-y}}{y}$ for $0<x<\infty$, $0<y<\infty$ and $f(x,y)=0$ otherwise.
(1) Find the conditional density of X, given that $Y=y$. (5%)
(2) Find $P\{X>1\mid Y=y\}=?$ (5%)

二、統計部分：50 分

1. (a) Let $X_1,X_2,\cdots,X_n$ be a random sample from a gamma distribution with parameters α and β, where $\alpha>0$ and $\beta>0$. Find the moment estimators of α and β. (5%)
(b) Let $X_1,X_2,\cdots,X_n$ denote a random sample from a Poisson distribution with mean θ. Find an UMVUE of $e^{-\theta}$. (10%)

2. Let $X_1,X_2,\cdots,X_n$ denote a random sample from the Negative Exponential distribution with p.d.f. $f(x;\theta)=\dfrac{1}{\theta}e^{-x/\theta}$, $x>0,\theta>0$. Find the ML estimator of the reliability function $R(x)=P(X_1>x)$. (10%)

3. Let $X_1,X_2,X_3,\cdots,X_{30}$ be a random sample of size 30 from an exponential distribution with mean $\beta=10$. Use the Central Limit Theorem to find an approximation for b, where $P\left[\overline{X}<b\right]=0.95$. (10%)

4. (a) A vice-president in charge of sales for a large corporation claims that salespeople are averaging no more than fifteen sales contacts per week. As check on his claim, $n=36$ salespeople are selected at random, and the number of contact is recorded for a single randomly selected week. The sample reveals a mean of seventeen contacts and a variance of 9. Does the evidence contradict the vice-president's claim? Use $\alpha=0.05$. (7%)
(b) Suppose that the vice-president in interested in testing $H_0:\mu=15$ against $H_a:\mu=16$. With the data as given in (a), find the probability of a type II error. (8%)

Table: Area $\Phi(x)$ under the standard normal curve to the left of x

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990