

國立嘉義大學 99 學年度
應用數學系碩士班 (甲組) 招生考試試題

科目：微積分

說明：本考試試題為計算、證明題，請標明題號，同時將過程作答在「答案卷」上。
(第 1~10 題每題 10 分，共 100 分)

1. Evaluate each of the following limits, if it exists.

(a) $\lim_{x \rightarrow 2} \frac{x^4 - 2x^3 - x^2 + 3x - 2}{x - 2}$. (5%) (b) $\lim_{x \rightarrow 0} \frac{\sin(8x)}{3x}$. (5%)

2. Find an equation of tangent line to the curve $x^2y + (y+x)^3 = 1$ at the point $(-1, 1)$. (10%)

3. Let $f'(x) = \frac{1}{\sqrt{1+x^4}}$, $x \in \mathbb{R}$, and g be the inverse function of f . Show that g satisfies $g''(x) = 2g^3(x)$. (10%)

4. (a) Find the Taylor series for f centered at 2 if $f^{(n)}(2) = \frac{(-1)^n n!}{5^n (n+1)}$, $n = 0, 1, 2, 3, \dots$. (5%)

(b) What is the radius of convergence of the Taylor series? (5%)

5. Evaluate the double integral $\iint_R (x+y) dy dx$, where the region

$$R = \{(x, y) \mid x \geq 0, x^2 + y^2 = r^2, r > 0\}. \quad (10\%)$$

6. Evaluate the integral.

(a) $\int_0^1 \int_0^1 e^{\max\{x^2, y^2\}} dy dx$. (5%) (b) $\int_0^4 \int_{\sqrt{x}}^2 \sqrt{1+y^3} dy dx$. (5%)

7. Let $f(x) = \frac{x(1+x)(2+x)\cdots(10+x)}{(1-x)(2-x)\cdots(10-x)}$. Compute $f'(0)$. (10%)

8. Find the absolute maximum and minimum values of $f(x, y) = x^2 + 3y^2 + 2y$ on the unit disk $x^2 + y^2 \leq 1$. (10%)

9. Find the interval of convergence of $\frac{x}{2^2 \cdot 3} + \frac{x^2}{3^2 \cdot 3^2} + \frac{x^3}{4^2 \cdot 3^3} + \dots$. (10%)

10. Find the equations of the tangent plane and the normal line to the surface $3z^y = yz + 2x^z$ at the point $(1, 1, 1)$. (10%)