# 國立嘉義大學九十五學年度
# 生物機電工程學系碩士班招生考試試題
## 科目：工程數學

1. The Laplace transform of function $f(t) = \sin bt$ is defined as

   $L[\sin bt] = \dfrac{b}{s^2 + b^2}$. Please obtain this result by the following two

   approaches: (25%)

   (a) Use the definition of Laplace transform $L[f(t)] = \int_0^{-\infty} f(t)e^{-st}dt$.

   (b) Derive the Laplace transform for a periodic function,

   $f(t) = f(t+T)$, where $T$ is the fundamental period. Then, apply

   your result to the function $f(t) = \sin bt$.

2. Consider the predator/prey model $\begin{Bmatrix} x' \\ y' \end{Bmatrix} = \begin{bmatrix} 6x - 3xy \\ xy - 12y \end{bmatrix}$: (25%)

   (a) Determine the critical points and classify their types and

   properties of stability.

   (b) Obtain the trajectories of the predator/prey system.

3. A steady fluid moves through a space with a velocity vector

   $\vec{F} = (4x^3 + 7y + 2z^3)\vec{i} + (3x^2 - 12x^2y + 2yz)\vec{j} + (x^2 + y^2 - z^2)\vec{k}$ (25%)

   (a) Evaluate the net outflow rate of $\vec{F}$ across a sphere

   $x^2 + y^2 + z^2 = 16$.

   (b) What is the outflow rate across the upper hemisphere

   $x^2 + y^2 + z^2 = 16, \quad z > 0$ and the lower hemisphere

   $x^2 + y^2 + z^2 = 16, \quad z < 0$, respectively

4. Evaluate the complex integral $\oint_C \left( \dfrac{z\exp(\pi z)}{z^4 - 16} + z\exp(\dfrac{\pi}{z}) \right) dz$, where $C$ is

   the ellipse $9x^2 + y^2 = 9$ (counterclockwise). (25%)