國立嘉義大學九十五學年度
資訊工程學系碩士班招生考試試題

科目：數學

1. Explain the Prim's method. (10%) And give the comparison between the Prim's and the Kruskal's methods. (5%)

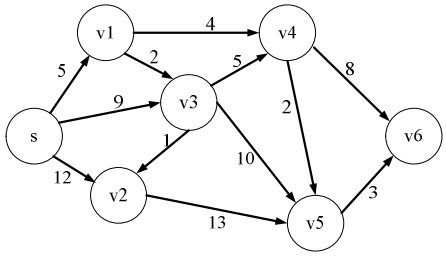
2. Show and explain the time complexities of the merge sort under two situations: the best case and the worst case. (10%)

3. What is the knapsack problem? (5%) What are the running times for the fractional knapsack problem and the general 0/1 knapsack problem, respectively? (5%)

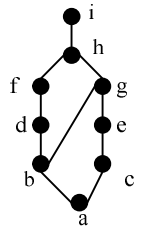
4. Input the following data in the given order: 13, 14, 15, 2, 1, 8, 3, 10, 7. And, show the following corresponding trees: (a) binary search tree；(5%) (b) 2-3 tree；(5%) (c) AVL tree. (5%)

5. Recurrence relations:
   (a) Solve the following recurrence relation：
   $a_{n+2} - 4a_{n+1} - 21a_n = 0, n \geq 0, a_0 = 0, a_1 = 1$. (6%)
   (b) Find the general solution for the recurrence relation：
   $a_n = -5a_{n-1} - 6a_{n-2} + 42 \cdot 4^n$, $n \geq 2$. (9%)

6. Use Dijkstra's algorithm to find the shortest paths from the vertex s to all the other vertices of the following graph. Show the running steps in detail. (15%)



7. Determine whether the following two posets are lattices. Explain your answer：(10%)
   (a) Hasse diagram:



   (b) The relation $\{(a,b) \mid a \text{ divides } b\}$ on the set {1,2,3,4,5}.

8. How many ordered pairs of integers (a,b) are needed to guarantee that there are two ordered pairs (a_1,b_1) and (a_2,b_2) such that $(a_1 \bmod 7) = (a_2 \bmod 7)$ and $(b_1 \bmod 6) = (b_2 \bmod 6)$. (10%)