

國立嘉義大學九十三年學年度
生物機電工程學系碩士班招生考試試題

科目：工程數學

1. Solve $(2x + 3y^2)dx = 2xydy$ (20%)
2. Given a matrix $A = \begin{bmatrix} 1 & -4 \\ 1 & 5 \end{bmatrix}$ to carry out the following two problems.
 - (a) Find the exponential matrix e^{At} . (10%)
 - (b) Solve the initial value problem $\vec{X}' = A\vec{X}$ with $\vec{X}(0) = \begin{Bmatrix} -2 \\ 3 \end{Bmatrix}$. (10%)
3. Evaluate $\oint_C \frac{z}{z^2 + 16} dz$, where C is the circle $|z - 2i| = 4$. (20%)
4. Find the volume of the solid that is bounded between the cones $z = \sqrt{x^2 + y^2}$ and $z = \sqrt{9x^2 + 9y^2}$ and the plane $z = 3$. (20%)
5. Solve the boundary value problem for the longitudinal elastic vibrations in a rod of length π whose left and right ends are both free. Assume $c = 1$ as the wave speed, and uniformly stretch the rod by one unit, releasing it without imparting any initial velocity. That is, the wave equation is $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ and the associated conditions are $\frac{\partial u}{\partial x}(0, t) = \frac{\partial u}{\partial x}(\pi, t) = \frac{\partial u}{\partial t}(x, 0) = 0$, and $u(x, 0) = \frac{x}{\pi}$ (20%)