國立嘉義大學九十三學年度應用數學系
碩士班考試試題

科目：線性代數（Linear Algebra）

說明：本考試試題為計算、證明題，請標明題號，同時將過程作答在「答案卷」上。

計算、證明題（1~2 題每題 20 分，3~6 題每題 15 分，共 100 分）

1. Let $A=\begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$. Find

   (a). the characterstic polynomial and the minimal polynomial of A. （6 分）

   (b). the eigenvalues and eigenspaces of A. （6 分）

   (c). an invertible matrix P such that $P^{-1}AP$ is diagonal and use it to find A^{10}. （8 分）

2. Let A be an $m\times n$ real matrix, B be an $n\times p$ real matrix. Prove that $rank(AB)\geq rank(A)+rank(B)-n.$ （20 分）

3. Let $P\in \mathrm{R}^{n\times n}$ be nonsingular and $A\in \mathrm{R}^{m\times n}$. Prove that the column space of AP is equal to the column space of A. In particular, AP and A have the same column rank. （15 分）

4. Let A and B be two $n\times n$ matrices. Show that $(AB-I)$ is invertible if $(BA-I)$ is invertible, where I is an $n\times n$ matrix. （15 分）

5. Let $A, B\in \mathrm{R}^{n\times n}$ be nonzero matrices. Prove that

   (a). if A and B are similar, then they have the same eigenvalues. (4 分)

   (b). if A is diagonalizable and its eigenvalues are all ± 1, then $A=A^{-1}$. (4 分)

   (c). if A is nilpotent, i.e., $A^k=0$, $\exists k\in \mathrm{N}$, then all of its eigenvalues are equal to zero. (3 分)

   (d). if A is nilpotent, then A is not diagonalizable. (4 分)

6. Find the value c so that the system of linear equations $\begin{cases} x+y+z=1 \\ x-y+z=2 \\ x+y-z=c \end{cases}$ has solutions in R^3, and in that case, find all the solutions. （15 分）