國立嘉義大學 100 學年度
應用數學系碩士班（甲組）招生考試試題

科目：線性代數

說 明 ：(1) 本考試試題為計算、證明題，請標明題號，同時將過程作答在「答案卷」上。
(2) 第 1~4 題每題 10 分，第 5~8 題每題 15 分，共 100 分。

1. Evaluate the determinant of the matrix $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$ and its rank. (10%)

2. Prove or disprove:

   (1) $\{(x,y) \mid x+2y=0\}$ is a subspace of $\mathbb{R}^2$. (5%)

   (2) $\{(x,y) \mid x+2y=1\}$ is a subspace of $\mathbb{R}^2$. (5%)

3. Let $S=\begin{bmatrix} 1 & 2 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix}$, $b=\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. Solve the matrix equation $Sx=b$ by Gauss-Jordan Elimination. (10%)

4. If $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is positive definite, prove that the inverse matrix of A is positive definite. (10%)

5. Let $A=\begin{pmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{pmatrix}$. Find an orthogonal basis for the column space of A. (15%)

6. Let u be a nonzero vector of $\mathbb{R}^n$ and I_n be an $n\times n$ identity matrix. Prove or disprove:

   (1) $I_n - \dfrac{2uu^t}{u^t u}$ is a symmetric matrix, where u^t is the transpose of the vector u. (7%)

   (2) $I_n - \dfrac{2uu^t}{u^t u}$ is an unitary matrix. (8%)

7. Let $A=\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$.

   (1) Find the null space and the column space of A. (8%)

   (2) Find all solution of $Ax=b$ where $b=\begin{bmatrix} 0 \\ 1 \\ 2 \\ 1 \end{bmatrix}$. (7%)

8. Let A be a $n\times n$ real matrix and let $e^{At}=I+At+\frac{1}{2}(At)^2+\cdots=\sum_{n=0}^{\infty}\frac{(At)^n}{n!}$ be the matrix exponential function. Compute

   (1) the 1st derivative $\frac{d}{dt}(e^{At})$ of function e^{At}. (7%)

   (2) If $\lambda_1, \lambda_2, \cdots, \lambda_n$ are n different eigenvalues of A. Find all eigenvalues of e^{At} (8%)