國立嘉義大學九十七學年度
管理研究所碩士班（乙組）招生考試試題

科目：應用微積分

1. Differentiate the following functions:（每題 6 分，共 30 分）

   a. $f(x) = \sqrt[4]{x} - \frac{3}{\sqrt{x}}$

   b. $f(x) = (2x+1)^2 (3-x)^3$

   c. $f(x) = \frac{(6-5x)^2}{(x^2-1)^2}$

   d. $f(x) = \frac{x^2}{e^{3x}}$

   e. $\ln\left[\left(\ln x^2\right)^5\right]$

2. When toasters are sold for p dollars apiece, local consumers will buy $D(p) = \frac{8,000}{p}$ toasters a month. It is estimated that t months from now, the price of the toasters will be $p(t) = 0.06t^{3/2} + 22.5$ dollars. Compute the rate at which the monthly demand for the toasters will be changing with respect to time 25 months from now.（10 分）

3. If the cost of a commodity is $C(x) = \frac{1}{8}x^2 + 3x + 98$ dollars when x units are produced and the selling price is $p(x) = \frac{1}{3}(75-x)$ dollars per unit, find the level of production where profit is maximized.（10 分）

4. The production function is $f(X) = 20X - X^2$ and the price of output is normalized to 1. Let ω be the price of the X-input. We must have $X > 0$.（15 分）

   a. What is the first-order condition for profit maximization if $X > 0$?

   b. For what values of ω will the optimal X be 10 ?

   c. What is the profit function ?

5. （20 分）Let

   $$Z = X_1 X_3 + X_1^2 - X_2 + X_2 X_3 + X_2^2 + 3X_3^2$$

   a. Find the first-order and second-order conditions.   b. Find the extreme value.

6. State with brief reasons whether the following statements are true, false or uncertain.（15 分）

   a. If a function is continuous in its domain, then we may conclude that it is differentiable at every point in its domain.

   b. The inverse-function rule which means that the derivative of the inverse function is the reciprocal of the derivative of the original function is applicable only when the function involved is strictly increasing.