國立嘉義大學九十六學年度
轉學生招生考試試題

科目：微積分

1. Differentiate the following functions:（每題 6 分，共 30 分）

   a. $f(x) = 2x^3 - \sqrt[3]{x} + \frac{4}{x^2}$

   b. $f(x) = \ln(e^{-x} + x)$

   c. $f(x) = 2(3x+1)^4(5x-3)^2$

   d. $f(x) = \left(\frac{x+2}{2-x}\right)^3$

   e. $f(x) = \ln(2x^3 - 5x + 1)$

2. Evaluate the following: （每題 5 分，共 30 分）

   a. $\lim_{x\to 0^+} \sqrt{x\left(1+\frac{1}{x^2}\right)}$

   b. $\lim_{x\to -\infty} \frac{x^4 + 3x^2 - 2x + 7}{x^3 + x + 1}$

   c. $\int \frac{x}{\sqrt{2x+1}}\,dx$

   d. $\int_{-1}^{2} 30(5x-2)^2\,dx$

   e. $\int \frac{\ln(3x)}{x}\,dx$

   f. $\int_{1}^{+\infty} \frac{x^2}{x^3+2}$

3. （每題 10 分，共 20 分）

   a. Find $\frac{dy}{dx}$ by implicit differentiation:

   $5x - x^2y^3 = 2y$

   b. Please sketch the graph of function $f(x) = x^4 + 4x^3 + 4x^2$. Be sure to show all key features such as the intervals of increase and decrease, intervals of concavity, intercepts, high and low points, points of inflection.

4. A manufacturer can produce radios at a cost of $5 apiece and estimates that if they are sold for *x* dollars apiece, consumers will buy (*20-x*) radios a day. At what price should the manufacturer sell the radios to maximize profit?（20 分）