國立嘉義大學九十二學年度轉學生招生考試試題

科目：高等微積分　　(請標明題號，並將計算過程寫在答案卷上)

1. (a). Show that $\lim_{(x,\,y)\to(0,\,0)} \dfrac{x^3 y^2}{(x^2+y^4)^2}$ does not exist. (10%)

   (b). Show that $\lim_{(x,\,y)\to(0,\,0)} \dfrac{y^5 - 3x^4 y}{x^4+y^4} = 0$. (10%)

2. Determine whether the following functions are uniformly continuous on $\Re$ and justify your answer.

   (a). $f(x) = x^2,\ \ x \in \Re$. (10%)

   (b). $f(x) = 3\sin x - 4\cos x,\ \ x \in \Re$. (10%)

3. Let $f(x) = \sum_{n=1}^{\infty} \dfrac{1}{(x+n)^2}$.

   (a). Show that f is a continuous function on $[0, \infty)$. (10%)

   (b). Evaluate $\int_0^1 f(x)\, dx$. (10%)

4. Let $h(x)$ be bounded and Riemann integrable on $[0, 1]$.

   (a). Show that if $h(x) > 0$ for all $x \in [0, 1]$, then $\int_0^1 h(x)\, dx > 0$. (10%)

   (b). Let $u(x) = \int_0^x h(t)\, dt$, $x \in [0, 1]$. Show that $u(x)$ is continuous on $[0, 1]$. (10%)

5. For $n = 1, 2, 3, \cdots$, put $f_n(x) = \dfrac{x}{1+nx^2}$, $x \in [0, 1]$.

   (a). Show that $\{ f_n \}$ converges uniformly on $[0, 1]$. (10%)

   (b). Let $f(x) = \lim_{n\to\infty} f_n(x)$, $x \in [0, 1]$. Is $f'(x) = \lim_{n\to\infty} f_n'(x)$ always true? Justify your answer. (10%)