國立嘉義大學100學年度
資訊工程學系碩士班（乙組）招生考試試題

科目：工程數學

(注意事項：1.不可使用計算機。 2.依次序作答。 3.試題隨試卷繳回。)

1. Solve $y'(x^2+4) = y^2$. (10%)

2. Solve by substitution: $x^2 y' = y^2 + xy,\ y(1) = 1/3$. (10%)

3. Solve $y'' + 9y = 27x(\cos 3x)/5$. (10%)

4. Evaluate the following problems:
   (a) The Laplace transform of $f(t) = 2e^{t}\cosh 3t$. (6%)
   (b) The Laplace transform of $g(t) = u(t-\pi)$, where $u(t)$ is the unit step function. (7%)
   (c) The inverse Laplace transform of $H(s) = \dfrac{3s+2}{s^2+4s+29}$. (7%)

5. Find the Fourier series of the given function $f(x)$, which is assumed to have the period 2π, where $f(x) = x$ if $-\pi < x < 0$, $f(x) = 1$ if $0 < x < \pi$. (10%)

6. Consider the temperature distribution $u(x, t)$ on a laterally insulated rod of length π governing by the heat equation $\dfrac{\partial u}{\partial t} = c^2 \dfrac{\partial^2 u}{\partial x^2}$ with thermal diffusivity $c^2 = 1$. If the boundary conditions at the ends are $u(0,t) = u(\pi,t) = 0$ and the initial condition is $u(x,0) = f(x) = 100\sin(2x)$, find the temperature distribution $u(x, t)$. (10%)

7. Given two matrices $[A] = \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{bmatrix}$; $[B] = \begin{bmatrix} 4 & -1 & -2 \\ 2 & 1 & 2 \\ 1 & 1 & -1 \end{bmatrix}$.
   (a) Find the inverse of $[A]$. (10%)
   (b) Find all the eigenvalues and their corresponding eigenvectors of $[B]$. (10%)

8. Use the Gram-Schmidt orthogonalization process to construct a set of orthonormal basis for R^3 from the basis vectors $\vec{v}_1 = (1,1,1)$, $\vec{v}_2 = (1,2,2)$, $\vec{v}_3 = (1,1,0)$. (10%)