國立嘉義大學 100 學年度

應用數學系碩士班（乙組）招生考試試題

科目：機率統計

說明：(1)本試題有機率、統計二大部分，各佔 50 分。
(2)本試題為計算、證明題，請標明每部分的題號，同時將過程作答在「答案卷」上。
(3)統計部分，計算過程如有需要常態曲線下之面積數值，請參考題目後面之附表。

一、機率部分：

1. Let the joint p.d.f. of X_1 and X_2 be

$$f(x_1,x_2)=\begin{cases}x_1+x_2, & 0<x_1<1, 0<x_2<1\\ 0 & ,\text{elsewhere}\end{cases}$$

(1) Find the conditional density $f(x_2 \mid x_1)$. (7%)
(2) Are X_1 and X_2 independent variables? Why? (8%)

2. Show that the random variable X is distributed as Bernoulli with the success probability of p if and only if $E(X^n)=p$, $n=1,2,\cdots$. (10%)

3. Suppose that X has the geometric distribution with parameter p, then $P(X>=$ where n . (10%)

4. Let X and Y be two random variables defined on the sample space and suppose that Y has finite variance. To show $Var($. (15%)

二、統計部分：

1. Let $X_1, X_2, \cdots, X_n$ be a random sample from an exponential distribution with probability density function

$$f(x)=e^{-(x-\theta)},\ x>\theta \quad ,$$

where θ is a location parameter of the distribution.
(1) Find the maximum likelihood estimator (MLE) of θ. (8%)
(2) Find the moment estimator of θ. (7%)

2. Consider the problem of testing $H_0:\mu \geq 400$ versus $H_1:\mu<400$ with standard deviation $\sigma=30$ and at the significance level $\alpha=0.02$. The probability of type II error of test is $\beta=0.1$ at the alternative μ_1=385. Find the sample size based on α and β risks. (10%)

3. If $X_1, X_2, \cdots$ are independent r.v.'s, to construct the MP test of the hypothesis *H* that the common distribution of the X's is Binomial(1,1/2) against the alternative *A* that it is Binomial(1,2/3) at level of significance $\alpha=0.05$. (10%)

4. Let $X_1, X_2, \cdots, X_n$ be random sample from uniform distribution with parameter θ, i.e. $X_i \sim U(\theta,2\theta)$, $i=1,2,\cdots$. There are two estimators of

$T_1=\frac{n+1}{2n}$, $T_2=\frac{n+1}{5n}(2)$

where $X_{(1)}=\min\{,,,\}$ … and $X_{(n)}=\max\{,,,\}$ … . Show that T_1 and T_2 are unbiased estimators of θ and T_2 is an efficient estimator. (15%)

Table: Area $\Phi(x)$ under the standard normal curve to the left of *x*

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990