國立嘉義大學九十七學年度
應用數學系碩士班招生考試試題

科目：線性代數

說明：本考試試題為計算、證明題，請標明題號，同時將過程作答在「答案卷」上。

（1~4 題每題 10 分，5~8 題每題 15 分，共 100 分）

1. To find a matrix $A \in \Re^{3\times 3}$ such that for all $x, y, z \in \Re$,

$$A\begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \,\middle|\, x - 2y + 3z = 0 \right\}. \quad (10\%)$$

2. Let $T: \Re^2 \to \Re^2$ be the linear transformation given by $T(x, y) = (x + 3y, 2x + y)$. Compute the adjoint T^* of T. Is T self-adjoint? (10%)

3. Prove that $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ and $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ are not similar. (10%)

4. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & a \end{bmatrix}$, where $a \in \Re$. Prove that there exists uniquely one value of a such that $\det(A) = 0$. (10%)

5. Let n be an integer and let $t_0, t_1, \cdots, t_n$ be distinct real numbers. Show that the set $\left\{ \phi_j(x) = \prod_{i=0,\, i\neq j}^{n} \frac{x - t_i}{t_j - t_i} \,\middle|\, j = 0, 1, \cdots, n \right\}$ is a basis for $\mathrm{P}_{\mathrm{n}}(\Re)$. (15%)

6. Find the Fourier coefficients of the vector $x^2 + x + 1$ with respect to the orthogonal set of vectors $\left\{ 1, x, x^2 - \frac{1}{3} \right\}$. (15%)

7. Let $A = \begin{bmatrix} 0 & 0 & 0 & a_1 \\ 0 & 0 & a_2 & 0 \\ 0 & a_3 & 0 & 0 \\ a_4 & 0 & 0 & 0 \end{bmatrix}$, where $a_1, a_2, a_3, a_4 \in \Re$ and let $a_1 a_4 > 0$ and $a_2 a_3 > 0$. Show that A is diagonalizable. (15%)

8. Find the Jordan canonical forms of $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 3 & 2 & 1 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$. (15%)