

國立嘉義大學 99 學年度
應用經濟學系碩士班招生考試試題

科目：統計學

1. Consider the simple regression model

$$y_i = \alpha + \beta x_i + \mu_i,$$

where $\mu_i \sim N(0, \sigma^2)$.

Decide whether the following statements are correct or incorrect? If incorrect, change the statement to make it correct. (30%)

- (a) The sample mean of the residuals is one.
- (b) The independent variable x_i and residual $\hat{\mu}_i$ are correlated.
- (c) The predicted value of y_i and residual $\hat{\mu}_i$ are correlated.
- (d) If the unit of measurement of y_i is changed, then it will affect the coefficient of determination R^2 .
- (e) The larger the sample size, the larger the covariance of the least squares estimators.
- (f) The variance of the forecast error is larger when the uncertainty in the simple regression model is smaller.

2. Consider the following log-linear model

$$\ln(y_i) = \alpha + \beta x_i + \mu_i,$$

where $\mu_i \sim N(0, \sigma^2)$. (30%)

- (a) What is the predicted value of y_i in the above model?
- (b) Construct a 95% prediction interval for y_i .

3. Let X be a random variable whose moment generating function is

$$M_X(t) = e^{-\lambda(1-e^t)}, \quad -\infty < t < \infty. \quad (20\%)$$

- (a) Find the probability distribution of X.
- (b) Determine the mean and variance of X.

4. Suppose $x_1, x_2, \dots, x_n \sim \text{i.i.d. } N(\mu, \sigma^2)$, where μ is the mean and σ^2 is the variance. Define

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i \quad \text{and} \quad s^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2. \quad (20\%)$$

- (a) Find the mean-squared error of \bar{x} .
- (b) Find the mean-squared error of s^2 .