國立嘉義大學九十六學年度

應用數學系碩士班招生考試（乙組）試題

科目：機率統計

說明：(1)本試題有機率、統計二大部分，各佔 50 分。
(2)本試題為計算、證明題，請標明每部分的題號，同時將過程作答在「答案卷」上。
(3)統計部分，計算過程如有需要常態曲線下之面積數值，請參考題目後面之附表。

一、機率部分：50 分

1. A random variable X whose probability density function is given, for some $\lambda>0$, by

$$f(x)=\begin{cases}\lambda e^{-\lambda x}, & x\ge 0\\ 0, & otherwise\end{cases}.$$

(a) What is the cumulative distribution function of X？ （5 分）
(b) Compute the expected value $E(X)$. （5 分）
(c) Compute the variance $Var(X)$. （5 分）

2. If X is a continuous random variable with probability density function (pdf) as follows:

$$f(x)=\frac{1}{\sqrt{2\pi}}e^{-x^2/2},\ -\infty<x<\infty.$$

(a) Find the pdf of $Y=X^2$ and identify the distribution of Y. （5 分）
(b) Find the moment generating function $M(t)$ of X. （5 分）

3. Let $Y_1<Y_2<Y_3<Y_4<Y_5$ be the order statistics of 5 independent observations from an exponential distribution that has a mean of $\theta=2$.
(a) Find the probability density function of the sample median Y_3. （5 分）
(b) Compute the probability that Y_4 is less than 5. （5 分）
(c) Determine $P(1<Y_1)$. （5 分）

4. Let $X_1, X_2, \Lambda\ , X_n$ denote a random sample of size n from a Poisson distribution with mean θ. Find the conditional expectation $E(X_1+3X_2+5X_3\mid\sum_{i=1}^{n}X_i)$. （10 分）

二、統計部分：50 分

1. Briefly answer each question below:
(a) Given a data set, how could you decide if the distribution of the data was approximately normal? Describe two methods. （4 分）
(b) What is meant by the power of a statistical test? How can the power of a test be increased? Describe two methods. （7 分）
(c) What are the assumptions for regression analysis? （4 分）

2. Let $X_1, X_2, X_3, \Lambda\ , X_{16}$ be identical and independent normal variables, each with mean μ and variance 1. We intend to test $H_0:\mu=10$ against $H_1:\mu>10$. The basic decision rule is that we will reject H_0 if sample mean $\overline{X}>c$.
(a) Find $c=?$ for a significant level of 0.05. （5 分）
(b) What is the probability of a type II error when $\mu=11$? （5 分）

3. (a) Let $X_1, X_2, \Lambda\ , X_n$ denote a random sample of size n from the binomial distribution $B(1,\theta)$ with $0\le\theta\le 2/3$. Find the maximum likelihood estimator (MLE) of θ. （5 分）
(b) Let $X_1, X_2, \Lambda\ , X_m$ denote a random sample of size m from the binomial distribution $B(n,\theta)$ and $X\sim B(n,\theta)$. Find the uniformly minimum variance unbiased estimator (UMVUE) of $P(X<1)$ based on $X_1, X_2, \Lambda\ , X_m$. （10 分）

4. Let $X_1, X_2, \Lambda\ , X_n$ denote a random sample of size n from an exponential distribution with probability density function

$$f(x)=e^{-(x-\theta)},\ x>\theta.$$

Find the $100(1-\alpha)\%$ confidence interval for θ of minimum length. （10 分）

背面尚有附表

Table: Area $\Phi(x)$ under the standard normal curve to the left of x

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990