國立嘉義大學九十三學年度
資訊工程學系碩士班招生考試試題

科目：數學

一、

(1) Please connect the a, b, c,…,h to 1,2,3,…,8: there exists one-to-one mapping relationship.

a. Identity:	1. $(p\vee q)\vee r \Leftrightarrow p\vee(q\vee r)$, $(p\wedge q)\wedge r \Leftrightarrow p\wedge(q\wedge r)$
b. Domination:	2. $p\vee q \Leftrightarrow q\vee p$, $p\wedge q \Leftrightarrow q\wedge p$
c. Idempotent:	3. $\neg(\neg p) \Leftrightarrow p$
d. Double Negation:	4. $p\vee p \Leftrightarrow p$, $p\wedge p \Leftrightarrow p$
e. Commutative:	5. $p\vee \mathrm{T} \Leftrightarrow \mathrm{T}$, $p\wedge \mathrm{F} \Leftrightarrow \mathrm{F}$
f. Associative:	6. $p\wedge \mathrm{T} \Leftrightarrow p$, $p\vee \mathrm{F} \Leftrightarrow p$
g. Distributive:	7. $\neg(p\vee q) \Leftrightarrow \neg p \wedge \neg q$, $\neg(p\wedge q) \Leftrightarrow \neg p \vee \neg q$
h. De Morgan's:	8. $p\vee(q\wedge r) \Leftrightarrow (p\vee q)\wedge(p\vee r)$, $p\wedge(q\vee r) \Leftrightarrow (p\wedge q)\vee(p\wedge r)$

a_?_ , b_?_ , c_?_ , d_?_ , e_?_ , f_?_ , g_?_ , h_?_ (10%)

(2) Clock (Modular) Arithmetic: Find an integer x where $0\le x\le 28$ such that $(3^{64} - x)$ is a multiple of 29. (6%)

(3) Clock (Modular) Arithmetic: Find an integer y where $0\le y\le 36$ such that $(2^{36} + 2 - y)$ is a multiple of 37. (4%)

二、

(1) Let three sets A={1,2,3,4}, B={a, b, c}, C={w, x, y, z} have the mapping relationship as:

f：A→B, g：B→C;

f = { (1 , b) , (2 , a) , (3 , b) , (4 , c) };

g = { (a , y) , (b , z) , (c , x) };

What is $\boldsymbol{w}$, if $g(f(\boldsymbol{w})) = z$? (5%)

(2) Let ＜$\boldsymbol{R}$, + , •＞ be the expression of a "ring". A subring is a subset $\boldsymbol{S}$ of $\boldsymbol{R}$ with the operations + and • of $\boldsymbol{R}$ restricted to $\boldsymbol{S}$ and such that $\boldsymbol{S}$ is a ring by itself. Now, if $\boldsymbol{S}$, $\boldsymbol{T}$ are the subrings of ring $\boldsymbol{R}$, prove or disprove that $\boldsymbol{S}\cap\boldsymbol{T}$ is also a subring of $\boldsymbol{R}$. (15%)

三、Please draw binary trees **T_5, T_6 and T_7** and show ***String5***, ***String6*** and ***String7*** according to the following table. In this problem, you are assigned to solve them using the given heuristic program statements as follows : (10%)

```
#include "MyBinaryTreeLib.h"
struct BinaryTree *MyBiTree[7];
char *Postorder(struct BinaryTree **Tree, char *Preorder, char *Inorder);
…
```

MyBiTree[7]：	**Preorder：**	**Inorder：**	**Postorder：**
T_1	xNeUYC	NxUYCe	NCYUex
T_2	xNeUYC	NxUCYe	NCYUex
T_3	xNeUYC	NxYCUe	NCYUex
T_4	xNeUYC	NxCYUe	NCYUex
T_5	E39ICS	39ECIS	***String5***
T_6	E39ICS	93ECIS	***String6***
T_7	E39ICS	39ESIC	***String7***

四、The definition of big-Oh notation is that a function $f(n)$ is $O(g(n))$ if there exist constants n_0 and c such for all values $n > n_0$, $f(n) < c * g(n)$.

(1) Assume that $f(n)$ is $O(g(n))$. Let $f_I(n) = \alpha * f(n)$, where α is a constant. Demonstrate that $f_I(n)$ is still $O(g(n))$. *Hint:* Find constants stated in the above definition. (5%)

(2) Assume that $f(n)$ is $O(g(n))$, where $f(n)$ and $g(n)$ are both functions of n, $g(n) > 1$ for all n. Demonstrate that $f(n) + \alpha$, for any constant α, is still $O(g(n))$. (5%)

五、

(1) Show the result after inserting 2, 1, 4, 5, 9, 3, 6 into an initially empty AVL tree. (10%)

(2) Insert 3, 1, 4, 6, 9, 2, 5 into an initially empty binary search tree. Show the result of deleting the root. (10%)

六、A pattern matching problem is to find the starting position of a pattern in a string. The time complexity of Knuth, Morris, Pratt pattern matching algorithm is O(m + n), where m is the length of string and n is the length of pattern. A failure function for a pattern is defined as below.

If $p = p_0p_1 \ldots p_{n-1}$ is a pattern, then its failure function, *failure*, is defined as:

$$failure(j) = \begin{cases} \text{largest } i < j \text{ such that } p_0 p_1 \ldots p_i = p_{j-i} p_{j-i+1} \ldots p_j & \text{if such an } i \ge 0 \text{ exists} \\ -1 & \text{otherwise} \end{cases}$$

The Knuth, Morris, Pratt algorithm is:

```
int match(char *s, char *pat)
{
        int i = 0, j = 0, lens = strlen(s), lenp = strlen(pat);
        while (i < lens && j < lenp) {
                if (s[i] == pat[j]) {
                        i++; j++; }
                else if (j == 0) i++;
                        else j = failure[j-1]+1;
        }
        return ((j == lenp) ? (i  –  lenp) : -1);
}
```

Given a string "abcaabbcaaaaaaa" and a pattern "abcabcacab".

(1) Compute the failure function for the pattern. (10%)

(2) What are the values of i and j variables in the algorithm after the statement "else j = failure[j-1]+1;" is executed for the first time? (10%)