國立嘉義大學九十三學年度
土木與水資源工程學系碩士班招生考試試題

科目：工程數學

注意：1.本試題不可使用計算機

2.本試題如條件不足，請自行假設

1. Find the general solution for the following equations. (20%)

(a) $xy'' - 3y' + 4\frac{y}{x} = 0$

(b) $y' + y = xy^4$

2. Prove $\begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is a orthogonal matrix and find it's Eigenvalues. (15%)

3. Use Convolution theorem of Laplace transform to solve the equation of

$$y(t) = e^{-2t} + \int_0^t y(\tau)e^{(t-\tau)}d\tau \qquad (15\%)$$

4. Use Laplace transform to solve the equation. (15%)

$$y'' - 3y' + 2y = 4e^{2t}, \quad (\ y(0) = -3, \quad y'(0) = 5)$$

5. Find the Fourier Series of function **f(t)**, the period **T**= $\frac{\pi}{2}$

$$\mathbf{f(t)} = \begin{cases} \mathbf{t} & \text{in} \quad -\frac{\pi}{8} < \mathbf{t} < \frac{\pi}{8} \\ (\frac{\pi}{4}\text{-}\mathbf{t}) & \text{in} \quad \frac{\pi}{8} < \mathbf{t} < \frac{3\pi}{8} \end{cases} \qquad (20\%)$$

6. Find the unit normal vector at point (1,1,2) on the surface of the equation

$\mathbf{z^2 = 2(\ x^2 + y^2\ )}$。(15%)