

國立嘉義大學九十七學年度  
數學教育研究所碩士班（一般生）招生考試試題

科目：微積分

1. Let the function  $f(x) = \begin{cases} x^2 - x, & \text{if } x \text{ is irrational} \\ m x + b, & \text{if } x \text{ is rational} \end{cases}$ , where  $m$  and  $b$  are

constants. Determine the constants  $m$  and  $b$  such that  $f$  is continuous and differentiable at 1. (10%)

2. Find the limit: (a).  $\lim_{x \rightarrow 0} \frac{1}{x} \int_0^{x^2} \sqrt{t} \sin t \, dt$ . (10%)

(b).  $\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x^2}$ . (10%)

3. Find the extreme values of the function  $f(x, y) = x^2 + y^2 - 2x + 2y + 3$  subject to the constraint  $x^2 + y^2 \leq 8$ . (20%)

4. Evaluate  $\int_0^{\infty} \frac{x^2}{(x^2 + 1)^2} dx$ . (10%)

5. Find the shortest distance from the point  $(-1, 1, 1)$  to the set of points given by the equation  $z = xy$ . (20%)

6. Let  $f(x) = \frac{3}{5 - 2x}$ . (20%)

(a) Write  $f(x)$  as a power series centered at 0, i.e.,  $f(x) = \sum_{n=0}^{\infty} a_n x^n$ .

(b) Determine the interval of convergence of this power series.