

國立嘉義大學 99 學年度
生物機電工程學系碩士班招生考試試題

科目：工程數學 (※禁止使用計算機)

1. Following equation is a linear nonhomogeneous equation.

$$y'' + 2y' - 3y = 2x; \quad y(0) = 3, \quad y'(0) = 3.$$

- (a) Solve the differential equation using the method of undetermined coefficients. (15%)
(b) Solve the differential equation using Laplace transform. (10%)

2. Consider the matrix $\mathbf{A} = \begin{bmatrix} 0 & 2 & 0 \\ 3 & 0 & -2 \\ -2 & 0 & 0 \end{bmatrix}$.

(a) Find the eigenvalues and eigenvectors of \mathbf{A} . (15%)

(b) Is \mathbf{A} diagonalizable? (10%)

3. Given a differential equation $x^2 y'' - 3xy' + 4y = 0$.

(a) Verify that $y_1 = x^2$ is a particular solution. (10%)

(b) Find $y_2 = ?$ by the method of the reduction of order. (15%)

4. Solve the wave equation for a string: (25%)

$$\text{PDE: } \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}, \quad 0 < x < 1, \quad t > 0.$$

$$\text{BC: } u(0, t) = 0, \quad u(1, t) = 0, \quad t > 0.$$

$$\text{IC: } u(x, 0) = 0, \quad \left. \frac{\partial u}{\partial t} \right|_{t=0} = \sin(\pi x), \quad 0 < x < 1.$$