國立嘉義大學九十一學年度轉學生招生考試試題

科目：高等微積分　　　（請標明題號，並將計算過程寫在答案卷上）

1. Given $f_n:[0,1]\to\Re$, $f_n(x)=x^2-x^n$, $\forall n\in\aleph$.

   (a). Find a function $f:[0,1]\to\Re$ so that $\lim_{n\to\infty} f_n(x)=f(x)$.（10%）

   (b). Does f_n converge uniformly to f ? Justify your answer.（10%）

2. Given $f(x,y)=\begin{cases}\dfrac{2xy}{x^2+y} & \text{if } x^2\neq -y\\ 0 & \text{if } x^2=-y\end{cases}$

   (a). Let $e=(e_1,e_2)$ be a unit vector. Evaluate the directional derivative of f at the point $(0,0)$ in the direction e.（10%）

   (b). Is f differentiable at the point $(0,0)$? Justify your answer.（10%）

3. (a). Prove that a set $A\subset\Re^n$ is compact iff it is closed and bounded.（10%）

   (b). Is the set $\bigcap_{n=1}^{\infty}\left[1+\dfrac{n}{e^n},\ 2+\dfrac{\sin n}{n}\right]$ compact?（10%）

4. (a). Suppose that $f:[a,b]\times[c,d]\to\Re$ is continuous and $\dfrac{\partial f}{\partial y}$ is continuous on $[a,b]\times[c,d]$.

   Show that $\dfrac{d}{dy}\displaystyle\int_a^b f(x,y)\,dx=\int_a^b \frac{d}{dy}f(x,y)\,dx$.（10%）

   (b). Let $t>0$. Calculate $\dfrac{d}{dt}\displaystyle\int_0^\infty e^{-tx}\,dx$.（5%）

   (c). Calculate $\displaystyle\int_0^\infty x^{10}e^{-x}\,dx$.（5%）

5. (a). Show that $\displaystyle\int_1^\infty \frac{\sin x}{x}\,dx$ is conditional but not absolute convergent.（10%）

   (b). Evaluate $\displaystyle\lim_{n\to\infty}\int_0^1 \frac{e^x\ \sin nx}{n}\,dx$.（10%）