國立嘉義大學九十一學年度轉學生招生考試試題

科目：工程數學　　　　　　　　　　　　（請將答案寫在答案卷上）

1. Solve the initial value problem: $y''+2y'+y = \delta(t-2)$ with $y(0) = 0$ and $y'(0) = 1$, where $\delta(t)$ is a unit impulse function. (25%)

2. The Fourier integral is defined as：$f(x) = \dfrac{1}{\pi}\int_{0}^{\infty}[A(\omega)\cos\omega x + B(\omega)\sin\omega x]d\omega$, where

   $A(\omega) = \int_{-\infty}^{\infty} f(x)\cos\omega x\, dx$, $B(\omega) = \int_{-\infty}^{\infty} f(x)\sin\omega x\, dx$.

   (1) Find the <u>Fourier integral</u> of $f(x) = \begin{cases} e^{-x}, & x > 0 \\ 0, & x < 0 \end{cases}$.

   (2) Find the value of the resulting integral at $x = 0$ and verify that $\displaystyle\int_{0}^{\infty}\frac{d\omega}{1+\omega^2} = \frac{\pi}{2}$　(25%)

3. If $A = \begin{bmatrix} 1 & 1 \\ -1 & 3 \end{bmatrix}$, find

   (a) the eigenvalues and generalized eigenvectors of A;

   (b) e^{At}. 　(25%)

4. Solve $\dfrac{\partial U}{\partial t} = \dfrac{\partial^2 U}{\partial y^2}$, subject to $U(0,t) = U(\pi,t) = 0, U(y,0) = \sin y$. 　(25%)