

國立嘉義大學九十三年度轉學生招生考試試題

科目：高等微積分

(請標明題號，並將計算過程寫在答案卷上)

1. Evaluate (a) $\lim_{n \rightarrow \infty} \int_0^1 \frac{nx^n}{1+x} dx$. (10%)
(b) $\lim_{x \rightarrow 0} \frac{6\sin(x^2) + x^6 - 6x^2}{x^{10} - 5x^{11} - 6x^{12}}$. (10%)
2. Show that $f(x) = \sqrt{x}$ is a uniformly continuous function on $(0, \infty)$,
but $g(x) = \frac{1}{x}$ is not. (20%)
3. (a). Show that if K is a compact subset of \mathfrak{R} , then $\sup(K) \in K$. (10%)
(b). Prove or disprove that the inverse image of a compact set under a continuous function must be compact. (10%)
4. Let A be the region in \mathfrak{R}^3 bounded by $x \geq 0$, $y \geq 0$, $z = 2$ and the surface $z = x^2 + y^2$.
Evaluate $\iiint_A x dx dy dz$. (20%)
5. Let the function $\begin{cases} h(x) = x^2 \sin \frac{1}{x} & \text{if } x \neq 0 \\ h(x) = 0 & \text{if } x = 0 \end{cases}$.
(1). Calculate $h'(0)$. (10%)
(2). Show that $h'(x)$ is not continuous at 0. (10%)