國立嘉義大學 99 學年度
應用數學系碩士班（甲組）招生考試試題

科目：線性代數

說明：(1)本考試試題為計算、證明題，請標明題號，同時將過程作答在「答案卷」上。

(2)第 1~4 題每題 10 分，第 5~8 題每題 15 分，共 100 分。

1. Fill in the six entries: a, b, c, d, e, f in the following 4×4 matrix

$$\begin{bmatrix} 1 & -1 & a & 5 \\ b & 4 & c & 8 \\ 2 & -7 & -1 & d \\ e & f & 6 & 3 \end{bmatrix},$$

so that the matrix is symmetric. (10%)

2. Let A and B be two square matrices. Does

$$(A+B)^2 = A^2 + 2AB + B^2$$

hold? If so, prove it; if not, give a counterexample and state under what conditions the equation is true. (10%)

3. Let A be a $m\times n$ real matrix. Prove $A^t A$ is nonsingular iff A has linearly independent columns. (10%)

4. Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 2 & 0 \\ 0 & 1 & 0 \end{bmatrix}$. Find an orthogonal basis for the column space of A. (10%)

5. For vectors v and w in an inner product space. Prove that $v-w$ and $v+w$ are perpendicular if and only if $\|v\| = \|w\|$. (15%)

6. Find a diagonal 3 by 3 matrix D and an invertible 3 by 3 matrix P such that

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} = P^T DP. \text{ (15\%)}$$

7. Compute the inverse of $\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ by Gauss-Jordan elimination. (15%)

8. Let V, W be vector spaces, and suppose that $\{v_i\}_{i=1}^n$ is a basis for V. Prove that: for any $\{w_i\}_{i=1}^n \subset W$, there exists exactly one linear transformation $f: V \to W$ such that $f(v_i) = w_i$ for each $i \in \{1, \cdots, n\}$. (15%)