國立嘉義大學九十六學年度

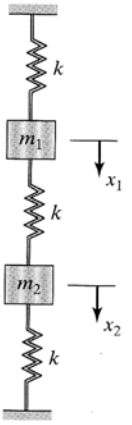
生物機電工程學系碩士班招生考試試題

科目：工程數學　　（※禁止使用計算機）

1. Consider a spring-mass system as shown in the Figure: (20%)

   (a) Derive the system of differential equations describing the straight-line vertical motion of the coupled springs.

   (b) Solve the system when $k=1$, $m_1=m_2=1$, $x_1(0)=1$, $x_1'(0)=0$, $x_2(0)=0$, $x_2'(0)=1$



2. Evaluate the followings: (30%)

   (a) the convolution $t * e^{2t} = ?$

   (b) the Laplace transform: $L[t^2 e^{-t}\cos 2t] = ?$

   (c) the inverse Laplace transform : $L^{-1}[\dfrac{1}{(s^2+3^2)^2}] = ?$

3. Solve each of the following matrix equations. *Hint : There may be no solution.* (20%)

   (a) $X^2 - 5X + 3I = \begin{bmatrix} 1 & -4 \\ 2 & -5 \end{bmatrix}$

   (b) $X^2 - 4X + 3I = \begin{bmatrix} 2 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

4. If $\vec{F}$ is a gradient field, find the potential function of $\vec{F}$ and evaluate $\int_A^B \vec{F}\cdot d\vec{R} = ?$ for the given points *A* and *B*. (20%)

   (a) $\vec{F} = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$; $A=(1,0,0)$, $B=(-1,0,\pi)$

   (b) $\vec{F} = (\dfrac{y}{z} - e^z)\vec{i} + (\dfrac{x}{z}+3)\vec{j} - (xe^z + \dfrac{xy}{z^2})\vec{k}$; $A=(-1,-1,1)$, $B=(0,\dfrac{e+1}{3},e)$

5. Find the steady-state temperature for a thin disk of radius *R* if the temperature on the boundary is $f(\theta)=\cos^2\theta$ for $-\pi \le \theta \le \pi$. *Hint :* $u(r,\theta) = A_0 + \sum_{n=1}^{\infty} r^n[A_n\cos(n\theta) + B_n\sin(n\theta)]$ (10%)