

國立嘉義大學 99 學年度
電子物理學系碩士班招生考試試題

科目：工程數學

1. Find all solutions if any of the following system of linear equations: (10%)

$$x_1 + 2x_2 - x_3 + x_4 = 2$$

$$2x_1 + x_2 + x_3 - x_4 = 3$$

$$x_1 + 2x_2 + 3x_3 + 2x_4 = 2$$

2. Find a series solution $y = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$ of the ordinary derivative equation $\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 6y = 0$, where n is a finite integer. (10%)

3. Find all the eigenvalues and their corresponding eigenvectors of the matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & i \\ 0 & i & 0 \end{bmatrix}$, where i is the imaginary unit. Normalize the eigenvectors. (20%)

4. Find the Fourier series of the function $f(t)$, which $f(t) = |t|$ for $-\pi < t < \pi$ and $f(t+2\pi) = f(t)$ for all t , and using the result calculate that $1 + \frac{1}{3^2} + \frac{1}{5^2} + \dots = ?$ (20%)

5. For the initial value problem $xy'' - 2y' + xy = 0$ subject to $y(0) = 0$, please answer the following questions. (20%)

(1) Use the Laplace Transform to find the solutions. No credit will be given with other method.

(2) What does the initial condition $y'(0)$ have to be? And how many solutions are there? Explain your reason.

6. Find the Laplace Transforms of $f(t)$ and $f'(t)$, where $f(t) \equiv \begin{cases} t^2, & 0 \leq t \leq 1, \\ 0, & t > 1. \end{cases}$ (20%)