國立嘉義大學 100 學年度
應用數學系碩士班（甲組）招生考試試題

科目：微積分

說明：本考試試題為計算、證明題，請標明題號，同時將過程作答在「答案卷」上。

1. Evaluate the following limits

   (a) $\lim_{n\to\infty}\dfrac{4\sqrt{n}+5\sqrt[4]{n}}{3\sqrt{n}+6\sqrt[4]{n}}$. (5%) (b) $\lim_{n\to\infty}\dfrac{\tan^{-1}n-\frac{\pi}{2}}{2n^{-1}}$. (5%)

2. Let $f(x)=\dfrac{x^3}{3^x},\ x>0$. Determine the intervals on which the function f is increasing or decreasing. (10%)

3. Let $f(c)=0$ and $f'(c)=12$. Find

   (a) $\lim_{h\to 0}\dfrac{f(c+h)}{3h}$. (5%) (b) $\lim_{h\to 0}\dfrac{f(c-2h)}{h}$. (5%)

4. Show that the function $u(x,t)=f(x+2t)+g(x-2t)$ satisfies the equation $\dfrac{\partial^2 u}{\partial t^2}(x,t)=4\dfrac{\partial^2 u}{\partial x^2}(x,t)$, where $f,g:R\to R$ are twice differentiable functions. (10 %)

5. Evaluate the following integrals.

   (a) $\int_1^e (\ln x)^2\,dx$ (5%) (b) $\int_0^\infty e^{-\frac{x^2}{2}}\,dx$. (Hint: $\int_0^\infty e^{-x^2}\,dx=\dfrac{\sqrt{\pi}}{2}$) (5%)

6. Find an equation of tangent line to the curve $3(x^2+y^2)^2=100xy$ at the point $(3,1)$. (10 %)

7. The natural logarithmic function can be defined by $\ln(x)=\int_1^x \dfrac{1}{t}\,dt,\ x>0$. Use this definition to prove that $\ln(ab)=\ln(a)+\ln(b)$ if $a>0$ and $b>0$. (10%)

8. Determine the convergence or divergence for each of the following series

   (a) $\sum_{n=1}^{\infty}\dfrac{e^{2n}}{n^n}$ (5%) (b) $\sum_{n=0}^{\infty}\dfrac{n^2 2^{n+1}}{3^n}$ (5%)

9. Let $f(x)=\dfrac{e^x-e^{-x}}{2},\ x\in R$ and g be the inverse function of f. Show that g satisfies $\dfrac{d}{dx}[g(x)]=\dfrac{1}{\sqrt{x^2+1}}$ (10%)

10. Evaluate the integral.

   (a) $\int_0^1\int_0^x e^{-x^2}\,dy\,dx$. (5%) (b) $\int_0^{\pi}\int_0^{\pi/2}\sin^2 x\cos^2 y\,dy\,dx$ (5%)