國立嘉義大學九十六學年度

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科目：工程數學

1. Find the Fourier transform of $f(x)$ $= \begin{cases} 1, & \text{either } -a-b < x < -a+b \text{ or } a-b < x < a+b \\ 0, & \text{otherwise} \end{cases}$ (where $a > b$). Fourier transform is defined by $\tilde{f}(k) = \dfrac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f(x)\, \mathrm{e}^{-\mathrm{i}kx} \mathrm{d}x$. Express your answer with product of trigonometric functions. （20%）

2. Find the solution of the differential equation

   $$x^2 y'' - xy' + y = x \lambda n(x)$$

   with the conditions: $y(1) = 0$ and $y'(1) = 0$（20%）

3. (a)Let $\vec{F} = f(r)\vec{r}$ and $f(1) = 1$, where $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ and $r = \sqrt{x^2 + y^2 + z^2}$
   Find $f(r)$ so that $\nabla \cdot \vec{F} = 0$ for $r \neq 0$.（10%）

   (b)Find $g(r)$ such that the flux of the vector field $\vec{V} = g(r)\vec{r}$ over the closed surface $x^2 + y^2 + z^2 = r^2$ is $2\pi r^2$.（10%）

4. Let $A = \begin{bmatrix} 0 & 2 & -2 \\ 0 & 1 & 0 \\ 1 & -1 & 3 \end{bmatrix}$, find $e^{At} = ?$（20%）

5. The temperature of a point (x_1, x_2, x_3) on the unit sphere $S = x_1^2 + x_2^2 + x_3^2 = 1$ is given by $T(x_1, x_2, x_3) = 1 + x_1x_2 + x_2x_3$. Using the method of Lagrange multipliers $\dfrac{\partial}{\partial x_i}(T + \lambda S) = 0$, find the temperature of the hottest point on the sphere.（20%）