

國立嘉義大學九十七學年度
應用經濟學系碩士班招生考試試題

科目：統計學

請清楚標明回答之題號，並請詳細列出計算過程。

一、 Y 為一服從 $n=3, p=1/3$ 之二項分配 (binomial distribution) 之隨機變數 (random variable)。(25%)

- (a) 請寫出 Y 之機率分配函數。
- (b) 請求算 $E(Y)$ 。
- (c) 請找出 Y 之動差母函數 (Moment-Generating Function)。
- (d) $Z \in [Y, Y+1]$, 為一均等分配 (uniform distribution), 求算 $E(Z|Y)$ 。
- (e) 求算 $E(Z)$ 。

二、設 X 與 Y 的聯合機率密度函數 (joint p.d.f.) 如下：(25%)

$$f(x, y) = \begin{cases} 2x, & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

- (a) 請求算 $f(y)$ 。
- (b) 試證明 X 與 Y 獨立。
- (c) 試求算 $p(X \leq Y)$ 。
- (d) 若 $U = X + Y$, 試求算 $f(u)$ 。
- (e) 試求算 $E(U)$ 。

三、Suppose that in a simple linear regression model, $Y_i = \beta_1 + \beta_2 X_i + \varepsilon_i$, we know that the intercept β_1 is equal to zero, that is, $\beta_1 = 0$. (21%)

- (a) Algebraically, what does the sum of squares function become?
- (b) Find a formula for estimating β_2 by using the least squares principle. This requires the use of calculus.
- (c) Repeat this exercise assuming that $\beta_2 = 0$, but that β_1 is not zero.

四、Indicate whether each of the following statements about the simple regression model, $Y_i = \beta_1 + \beta_2 X_i + \varepsilon_i$, is true or false. If false, explain why. (8%)

- (a) If the sample means of X and Y are zero, then the estimated Y -intercept is zero.
- (b) The slope of the simple regression model indicates how the actual value of Y changes as X changes.
- (c) The residuals from a least squares regression are all zeros.
- (d) If the sample covariance between X and Y is zero, then the slope of the least squares regression line is zero.

五、Professor E. Z. Stuff has decided that the least squares estimator is too much trouble. Noting that two points determine a line, Dr. Stuff chooses two points from a sample of size T and draw a line between them. The slope of this line he called the EZ-estimator of β_2 in the simple regression model. Algebraically, if the two points are (x_1, y_1) and (x_2, y_2) , the EZ-estimation rule is: $b_{EZ} = \frac{y_2 - y_1}{x_2 - x_1}$

Assuming that all the assumptions of the simple regression model hold: (21%)

- (a) Show that b_{EZ} is a “linear” estimator.
- (b) Show that b_{EZ} is an “unbiased” estimator.
- (c) Find the variance of b_{EZ} .